Lesson 08 Linear Programming Solutions

Solved Problem #1: see text book (use the graphical LP method)

Solved Problem #2a, d, e: see textbook (use the Maximize Excel Solver)

#1: Solve this problem using graphical linear programming and answer the questions for the following LP formulation.

\[
\text{Maximize } Z = 4X_1 + 3X_2 \\
\text{Subject to:} \\
\text{Material } 6X_1 + 4X_2 \leq 48 \text{lb} \\
\text{Labor } 4X_1 + 8X_2 \leq 80 \text{hr}
\]

a. Display the constraint equations on the same graph.

b. How many of each product should be made to maximize the objective function \( Z \)?
   
   \( 2 \) \( X_1 \)'s
   
   \( 9 \) \( X_2 \)'s

c. What is the maximum possible value of \( Z \)?
   
   \( 35 \)

d. Identify the binding constraints?

   Both, material and labor are bound because they are completely utilized (i.e. have no available slack)

#2: The manager of a production operation is considering linear programming to help him analyzed the scenario below where there are 2 products: A and B which are constrained by Material, Machinery, and Labor. Answer the following questions using graphical linear programming.
Maximize Revenue = 6A + 3B
Subject to:
Material 20A + 6B ≤ 600lb
Machinery 25A + 20B ≤ 1,000hr
Labor 20A + 30B ≤ 1,200hr

a. Explain why graphical linear programming is a feasible analysis tool.

Graphical LP is feasible because there are only 2 products in the mix. Therefore, it is easy to graph the constraints and determine the feasible solution space using a 2 dimensional graphical technique.

b. Display the constraint equations on the same graph.

c. How many of each product should be made to maximize the objective function Revenue?

24 A’s and 20 B’s

d. What is the maximum possible value of Revenue?

204

e. Identify the binding constraints?

Material and Machinery are bound because they are completely utilized (i.e. have no available slack)

f. If the manager decreases the Labor by 120 units, does that have an impact on the solution? Explain why he does not have to change the LP formulation to be able to answer this question.

No, even though there is a slack of 120 hours in Labor, the other constraints are bound; therefore, reducing the hours of Labor will not have an impact on the outcome.
g. After reviewing the original formulation solution, the manager decides to increase the Material and Machinery capacity by 10%. Why does this seem reasonable?

It is reasonable because the Material and Machinery constraints are binding production capacity. Since there is slack in the Labor constraint, the manager can possibly increase the maximum Revenue by increasing Material and Machinery capacity.

h. Using the information in g, discuss the impact the decision to increase Material and Machinery capacity has on the Revenue of the manager’s operation. What percentage increase does this decision have on company Revenue?

The quantity of S’s that can be produced increases from 24 to 26.4 units.
The quantity of B’s that can be produced increases from 20 to 22 units.
The total Revenue the company can make increases from $204 to $224.4, which is a 10% improvement.

#3: Solve the linear programming formulation using the appropriate Excel Solver template and answer the questions below.

Minimize Cost = 1.80S + 2.20T

Subject to:
Potassium 5S + 8T ≥ 200 grams
Carbohydrate 15S + 6T ≥ 240 grams
Protein 4S + 12T ≥ 180 grams
T ≥ 10 grams

a. Can you identify a business where this may be a valid scenario.

This could possibly be a company which is manufacturing foods which have minimum requirements for potassium, carbohydrate and protein requirements.

b. How many of each product should be made to minimize the objective function Cost?

8 S’s and 20 T’s

c. What is the minimum possible value of Cost?

58.4

d. Identify the binding constraints?

Potassium and Carbohydrates are bound because they have no available surplus (i.e. the minimum requirements for Potassium and Carbohydrates have been met for both products S and T).

#4: The owner of a small candy shop must decide how many packages (sold in one-pound bags) of deluxe mix (D) and how many packages of standard mix (S) to make and put on the display shelves to maximize her profit during the upcoming holiday season. The deluxe mix and standard mix use the same ingredients: peanuts (P) and raisins (R). The number of packages she can make is limited by the amount of the ingredients that she can obtain.
The deluxe mix contains .6 pound of raisins and .4 pound of peanuts. The standard mix has .5 pound of raisins and .5 peanuts. She is only able to buy 120 pounds of raisins and 120 pounds of peanuts for the entire season.

Peanuts cost $.60 per pound and raisins cost $1.50 per pound. The deluxe mix sells for $2.90 per pound and the standard mix sells for $2.25 per pound. The owner estimates that no more than 110 bags of either product can be sold.

Answer the following questions. Use the appropriate Excel Solver routine to perform the linear programming analysis.

a. What is the cost of a package of the deluxe mix? What is the cost of a package of the standard mix?

Cost of D = .6*cost of raisins + .4*cost of peanuts = .90 + .24 = $1.14
Cost of S = .5*cost of raisins + .5*cost of peanuts = .75 + .30 = $1.05

b. What is the profit contribution of a package of the deluxe mix? What is the profit contribution a package of the standard mix?

Profit of D = 2.90 - 1.14 = 1.76
Profit of S = 2.25 - 1.05 = 1.20

c. What is the objective function for Profit?

Profit = 1.76*D + 1.20*S

d. What are the constraint equations?

Raisins .6*D + .5*S <= 120 lbs
Peanuts .4*D + .5*S <= 120 lbs
D D <= 110 packages
S S <= 110 packages

e. How many packages of each mix should she produce to maximize Profit?

110 D’s and 108 S’s

f. What is the maximum Profit she can make during the holiday season off of the deluxe and standard mix products?

$323.20

g. The shop owner has not yet placed her order for the raisins and peanuts. Using the information in the linear programming output, how much of each ingredient should she order? Explain.

She should order 120 pounds of raisins and 98 pounds of peanuts. If she orders the available amount of 120 pounds of peanuts she will have 22 pounds left over.

h. Before the owner places her final order, she investigates to see if there are other sources where she can get the ingredients for the standard and deluxe products. She is able to find a vendor who can supply an additional 30 pounds of raisins at the same price. She is unable to find anyone who can supply any more peanuts. Before she places her final order she wants to see if any or all of the 30 pounds of raisins will help improve her profits.

i. How many pounds of raisins and peanuts should she order?
She should order 121 pounds of raisins and 99 pounds of peanuts.

ii. Given this new information, how many packages of each mix should she produce to maximize Profit?

\[ 110 \text{ D’s and 110 S’s} \]

iii. What is the maximum Profit she can make?

\[ \$325.60 \]

#5: A retired couple supplements their income by making fruit pies, which they sell to a local grocery store. During the month of September, they produce apple (A) and cherry (C) pies. The pies are sold to the grocer for the following prices: apple - $1.50 and cherry - $1.20.

Because of the high quality and fresh ingredients in the pies, the couple is able to sell all they can produce. In addition to the apples and cherries used in each pie, the pies use other ingredients which are the same: sugar and flour. For the month of September they have 1,200 cups of sugar (S) and 2,100 cups of flour (F). The couple working together can produce an apple pie in 6 minutes and a cherry pie in 3 minutes. Each apple pie contains 1.5 cups of sugar and 3 cups of flour. Each cherry pie requires 2 cups of sugar and 3 cups of flour.

If the couple plans to work no more than 60 hours, answer the following questions using graphical linear programming to maximize the Revenue the couple will make.

a. What is the objective function for Revenue?

\[ \text{Revenue} = 1.50 \times A + 1.20 \times C \]

b. What are the constraint equations?

- **Flour**  \[ 3A + 3C \leq 2,100 \text{ cups} \]
- **Sugar**  \[ 1.5A + 2C \leq 1,200 \text{ cups} \]
- **Labor**  \[ 6A + 3C \leq 3,600 \text{ minutes} \]

c. Display the constraint equations on the same graph.
d. How many of each pie should the couple produce to maximize Revenue?

**500 A’s and 200 C’s**

e. What is the maximum Revenue the couple can make during September?

**$990.00**

f. How much flour, sugar, and time will be unused?

**All flour and time are completely used. 50 cups of sugar will be left over.**

g. Assume the couple does not want to have any of the ingredients left over.

i. How much additional flour should the couple purchase?

**60 cups - making a total of 2,160 cups of flour**

ii. Assume the couple purchases the additional quantity of flour in g, what is their maximum Revenue? Under this scenario, how many pies should be made?

**$1,008**

**480 A’s and 240 C’s**

<table>
<thead>
<tr>
<th>-Products-</th>
<th>-Description-</th>
<th>Coefficients</th>
<th>Available Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Maximize</strong></td>
<td><strong>Flour</strong></td>
<td>3</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>Sugar</strong></td>
<td>1.5</td>
<td>2</td>
<td>1200</td>
</tr>
<tr>
<td><strong>Labor</strong></td>
<td>6</td>
<td>3</td>
<td>3600</td>
</tr>
</tbody>
</table>

**Optimal Values**

<table>
<thead>
<tr>
<th>A</th>
<th>C</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>240</td>
<td>1008</td>
</tr>
</tbody>
</table>

#6: A wood products company uses available time at the end of each week to make goods for inventory. Currently, two products are produced: a chopping board (C), and a knife holder (K). Both items require 3 operations: cutting, gluing and finishing. The manager of the company has collected the following information:

- Profit contribution: C - $2, K – $6
- Cutting: C – 1.4 minutes, K – .8 minutes
- Gluing: C – 5 minutes, K – 13 minutes
- Finishing: C – 12 minutes, K – 3 minutes

The manager has also determined that during the week there are 56 minutes available for cutting, 650 minutes available for gluing, and 360 minutes available for finishing.

Answer the following questions assuming that all items made for inventory will be sold. Use the appropriate Excel Solver routine to perform the linear programming analysis.

a. What is the objective function for Profit?
\[ \text{Profit} = 2C + 6S \]

b. What are the constraint equations?

- Cutting: \[ 1.4C + 0.8K \leq 56 \text{ minutes} \]
- Gluing: \[ 5C + 13K \leq 650 \text{ minutes} \]
- Finishing: \[ 12C + 3K \leq 360 \text{ minutes} \]

c. Determine the optimal quantities of each product to maximize \text{Profit}?

- 0 chopping boards (C) and 50 knife holders (K)

d. What is the maximum \text{Profit} that can be made by utilizing the excess time each week to make items for inventory?

- $300

e. Which resources are completely used? Which are not?

- Completely used: Gluing
- Not completely used: Cutting – 16 minutes; Finishing – 210 minutes

#7: The manager of the deli section of a grocery superstore has just learned that the department has 112 pounds of mayonnaise (M), of which 70 pounds is approaching its expiration date and must be used. To use up the mayonnaise, the manager has decided to prepare two items: a ham spread (H), and a deli spread (D). Each pan of ham spread will require 1.4 pounds of mayonnaise, and each pan of deli spread will require 1.0 pounds of mayonnaise.

The manager has on-hand an order for 10 pans of ham spread and 8 pans of deli spread. In addition, he would like to have at least 10 pans of each spread available for sale. Both spreads cost $3 per pan to make. The ham spread sells for $5 per pan, and the deli spread sells for $7 per pan. The manager wants to look at two alternatives.

First, he wants to look at the solution which will minimize \text{Cost}. Use the appropriate Excel Solver routine to perform the linear programming to minimize \text{Cost}?

a. What is the objective function for \text{Cost}?

\[ \text{Cost} = 3H + 3D \]

b. What are the constraint equations for minimizing \text{Cost}?

- Mayonnaise: \[ 1.4H + 1.0D \geq 70 \text{ lbs} \]
- H: \[ H \geq 20 \text{ pans} \]
- D: \[ D \geq 18 \text{ pans} \]

c. How many pans of each spread should the deli manager produce to minimize \text{Cost}?

- 37.14 pans of ham spread
- 18 pans of deli spread

d. What is the minimum \text{Cost}?
Second, he wants to determine the quantities that should be made to maximize Profit. Assuming the deli manager can sell all he produces, answer the following questions. Use the appropriate Excel Solver routine to perform the linear programming analysis to maximize Profit.

a. What is the objective function for Profit?

\[ \text{Profit} = 2H + 4D \]

b. What are the constraint equations for maximizing Profit?

Recall that the constraint equations for maximizing are \( \leq \) bounds; therefore,

- Mayonnaise \( 1.4H + 1D \leq 112 \text{ lbs} \)
- \( H \leq -20 \text{ pans} \)
- \( D \leq -18 \text{ pans} \)

c. How many pans of each spread should the deli manager produce to maximize Profit?

- 20 pans of ham spread
- 84 pans of deli spread

d. What is the maximum Profit?

\$376.00

Which solution should the manager implement?

He should implement the second solution to maximize Profit because it utilizes the 70 pounds of mayonnaise that is about to expire as well as meets the on-hand orders, and has at least 10 additional pans of each spread.