Lesson 17 – Inventory Management

An inventory is a stock or store of goods and inventory management focuses on the planning and control of finished goods, raw materials, purchased parts, work-in-progress and retail items.

Independent Demand

- A
- B(4)
- C(2)
- D(2)
- E(1)
- F(2)

Dependent Demand

- Independent demand is uncertain
- Dependent demand is certain

Different Types Of Inventory

Inventory must be managed at various locations in the production process:
- Raw materials or purchased parts
- Partially completed goods, called "work-in-progress (WIP)"
- Finished goods inventories (manufacturing organizations)
- Merchandise (retail organizations)
- Replacement parts, tools and supplies
- Goods-in-transit between locations (either plants, warehouses, or customers)
Inventory Management Locations

Receiving

Production Process

Finished Goods

Raw Materials

Work center

WIP

Work center

WIP

Work center

Inventory Management Locations

Functions Of Inventory Control

Inventory control is necessary to:

- Meet anticipated demand
- Smooth production requirements
- De-couple components of the production-distribution system
- Protect against stock outs
- Take advantage of order cycles
- Hedge against price increases or quantity discounts
- Permit operations to function smoothly and efficiently
- Increase cash flow and profitability

The inventory manager is constantly striving to manage inventory to the right level. If you have too much then you are taking away from your cash flow. If you have too little you may not be running your operations smoothly and efficiently and disappointing your customers.

Some Terminology

Some terms that common to inventory management are:

- **Lead time** - time interval between ordering receiving the order
- **Carrying (holding) cost** - the cost of holding an item for a specified period of time (usually a year), including cost of money, taxes, insurance, warehousing costs, etc
- **Ordering costs** - costs of ordering and receiving inventory
- **Shortage costs** - costs resulting when demand exceeds the supply of inventory on hand often resulting in down time, unsatisfied customers, and unrealized profits
- **Cycle counting** - a periodic physical count of a classification of inventory or selected inventory items to eliminate discrepancies between the physical count and the inventory management system
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Objectives Of Inventory Control

There are many objectives of inventory control. Simplistically they are to have the right amount (not too much – not too little) at the right place at the right time to maximize your cash flow, to have smooth efficient operations and to meet customer expectations.

- Inventory is money! Inventories must be managed “cost effectively” giving consideration to
  - Cost of ordering and maintaining inventory
  - Carrying costs
  - Timing of inventory to allow for smooth operations
- Inventory is necessary to meet customer requirements.
  Inventory must be managed to a required level of customer service
  - Ensure that the right product is produced at the right time to meet customer demand

Inventory Control Effectiveness

There are several measurements to determine how well a company manages its inventory. Industry information and specific competitor information can be obtained through industry associations and published financial reports.

- Days of inventory on hand - the amount of inventory on hand based on the expected amount of days of sales that can be supplied from the inventory
- Inventory turnover - the ratio of annual cost of goods sold to average inventory investment
- Customer satisfaction - quantity of backorders, percent of orders filled on time, customer complaints about delivery

Requirements For Effective Management

Some of the requirements for effective inventory management include:

- A system to keep track of the inventory on hand (raw materials, work-in-progress, finished goods, spare parts, etc).
- A system to manage purchase orders
- A reliable forecast of demand that includes the possible forecast error
- Knowledge of lead times and lead time variability
- Reasonable estimates of inventory holding costs, ordering costs, and shortage costs
- A classification system for inventory items

Integrated Management Information Systems are critical to the successful inventory manager. The inventory management system must be integrated with the financial, production, and customer service functions of the company.
Inventory Counting Systems

Inventory is such a major factor in a business operations that Counting Systems are necessary to ensure that inventory is being managed properly. Inventory systems are only as good as the information they contain ... and transaction errors can be very costly.

- Periodic system - physical counts made at periodic intervals throughout the year
- Perpetual (continual) inventory systems - keep track of additions and removals from inventory so that a continual running total of inventory on hand is available

Most inventory systems today utilize “bar codes” to accurately track inventory movements easily and cost effectively.

- Grocery stores
- Retail stores
- Auto rentals

Classification Systems

The ABC method provides for classification of inventory according to some measure of importance (usually by annual dollar usage)

- A - very important (accuracy within .2 percent)
- B - moderately important (accuracy within 1 percent)
- C - least important (accuracy within 5 percent)

The classification system does not necessarily mean that B and C items are unimportant from a production point of view. A stock-out of nuts and bolts which may be classified as C items can just as easily shut down a production line as a major component.

ABC Classification - Example

Example 1: Classify the inventory items below as A, B or C.

<table>
<thead>
<tr>
<th>Item</th>
<th>Annual Demand</th>
<th>Unit Cost</th>
<th>Annual Dollar Volume</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000</td>
<td>4,300</td>
<td>4,300,000</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>5,000</td>
<td>720</td>
<td>3,600,000</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>1,900</td>
<td>500</td>
<td>950,000</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>1,000</td>
<td>710</td>
<td>710,000</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>2,800</td>
<td>250</td>
<td>625,000</td>
<td>B</td>
</tr>
<tr>
<td>6</td>
<td>2,500</td>
<td>150</td>
<td>480,000</td>
<td>B</td>
</tr>
<tr>
<td>7</td>
<td>400</td>
<td>200</td>
<td>80,000</td>
<td>C</td>
</tr>
<tr>
<td>8</td>
<td>500</td>
<td>100</td>
<td>50,000</td>
<td>C</td>
</tr>
<tr>
<td>9</td>
<td>200</td>
<td>210</td>
<td>42,000</td>
<td>C</td>
</tr>
<tr>
<td>10</td>
<td>1,000</td>
<td>35</td>
<td>35,000</td>
<td>C</td>
</tr>
<tr>
<td>11</td>
<td>3,000</td>
<td>10</td>
<td>30,000</td>
<td>C</td>
</tr>
<tr>
<td>12</td>
<td>9,000</td>
<td>3</td>
<td>27,000</td>
<td>C</td>
</tr>
</tbody>
</table>
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How Much? When? To Order
How much and when to order depends on many factors including: ordering costs, carrying costs, lead times, variability in lead times, variability in demand, variability in production, etc.

The Economic Order Quantity (EOQ) is the order size (how much?) that minimizes the total cost of inventory.

The Reorder Point (ROP) is the inventory which triggers a reorder.

How Much? To Order – EOQ Models
There are 3 Economic Order Quantity (EOQ) models which can be used to determine how much to order. Each has a scenario under which it is appropriate. They are:

- Basic EOQ – instantaneous delivery
- EOQ – non-instantaneous delivery
- EOQ – quantity discount

Inventory Cycle – Instantaneous Delivery
Inventory instantaneously increases by the quantity (Q) received.
The Basic Economic Order – instantaneous delivery model assumptions are as follows:

1. Only one product is involved
2. Demand requirements are known
3. Demand is reasonably constant
4. Lead time does not vary
5. Each order is received in a single delivery ("instantaneously")
6. There are no quantity discounts

### Basic EOQ – Instantaneous Delivery Model

Carrying Cost = \( \frac{Q}{2} H \) where
- \( Q \) = Order quantity
- \( H \) = Holding (carrying) cost per unit

Annual Carrying Cost is linearly related to the Order Quantity

### Basic EOQ – Ordering Cost

Ordering Cost = \( \frac{D}{Q} S \) where
- \( Q \) = Order quantity
- \( D \) = Demand, usually in unit per year
- \( S \) = Ordering Cost

Ordering Cost decreases as Order Quantity increases; however not linearly
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### Basic EOQ – Total Cost

Total Cost \( (TC) = \text{Carrying Cost} + \text{Ordering Cost} \)

\[
TC = \frac{Q}{2} H + \frac{D}{Q} S
\]

### Basic EOQ – Instantaneous Delivery

The Basic Economic Order – Instantaneous delivery model EOQ is the quantity which minimizes Total Cost = Carrying Cost + Ordering Cost. It is where Carrying Cost = Order Cost and is calculated by:

**Basic EOQ**

\[
Q = \sqrt{\frac{2DS}{H}}
\]

**Length of Order Cycle**

\[
\frac{Q}{D}
\]

### Basic EOQ – Example

Example 2a: A local distributor for a national tire company expects to sell 9,600 steel belted radial tires of a certain size and tread design next year. Annual Carrying Cost is $16 per tire, and Ordering Cost is $75. The distributor operates 288 days per year. What is the EOQ?

**Economic Order Quantity**

\[
Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9,600)(75)}{16}} = 300
\]
Example 2b: How many times per year does the tire distributor reorder tires?

Number of Orders Per Year = \( \frac{D}{Q} \) = \( \frac{9,600}{300} \) = 32

Example 2c: What is the length of the order cycle (Cycle Time)?

Length of Order Cycle = \( \frac{Q}{D} \) = \( \frac{300}{9,600} \) = .03125

therefore
since there are 288 days in the year the
Order Cycle = .03125 * 288 = 9 days

Example 2d: What is the Total Annual Cost if the EOQ is ordered?

\[
TC = \frac{Q}{2}H + \frac{D}{Q}S = \frac{300}{2} \times (16) + \frac{9,600}{300} \times 75
\]

\[
= 2,400 + 2,400 = 4,800
\]
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Example 3a: Piddling Manufacturing assembles security monitors. It purchases 3,600 black and white cathode ray tubes (CRT’s) at $65 each. Ordering costs are $31, and annual carrying costs are 20% of the purchase price. Compute the optimal order quantity.

\[ S = \$31 \]
\[ C = 0.20(\$65) = \$13 \]

Economic Order Quantity \( Q = \sqrt{\frac{2DS}{H}} \)

\[ Q = \sqrt{\frac{2(3,600)(31)}{13}} = 131 \]

Example 3b: Compute the total annual ordering cost for the optimal order quantity.

\[ TC = \frac{Q}{2}H + \frac{D}{Q}S = \frac{131}{2}(13) + \frac{3,600}{131}(31) \]

\[ = \$852 + \$852 = \$1,704 \]
The basic EOQ model assumes instantaneous delivery; however, many times an organization produces items to be used in the assembly of products. In this case the organization is both a producer and user. Orders for items may be replenished (non-instantaneously) over time rather than instantaneously.

Consider the situation where a toy manufacturer makes dump trucks. The manufacturer also produces (production rate) the rubber wheels that are used in the assembly of the dump trucks. Let's consider 500 per day for example. In this case the ordering costs associated with an order for rubber wheels would be the cost associated with the setup and delivery of the rubber wheels to the dump truck assembly area. The manufacturer makes the dump trucks at constant rate per day (production rate). Let's consider 200 per day for example.

The inventory picture in this case is much different from the “saw-tooth” pattern we saw in the instantaneous model as shown on the next slide.
As you see in this example, the inventory (shown in yellow) depends on the production rate (shown in blue) and the usage rate (shown in pink).

How much to order depends on setup costs and carrying costs. The Economic Order Quantity (EOQ) is the order size that minimizes the total cost of inventory. Sometimes this is referred to as the Economic Run Quantity because it is dependent on the cumulative manufacturing production quantity.

Total cost = Carrying costs + Setup costs.

A schematic of the non-instantaneous considerations are shown on the next slide.
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**EOQ – Non-instantaneous Delivery Model**

Economic Run Quantity: \( Q_0 = \sqrt{\frac{2DS}{p - u}} \)

- \( p \): production rate
- \( u \): usage rate
- \( S \): ordering cost
- \( D \): annual demand

Maximum Inventory: \( I_{\text{max}} = \frac{Q_0}{p} (p - u) \)

Average Inventory: \( \frac{I_{\text{max}}}{2} \)

**Minimum Total Cost**

Minimum Total Cost: \( TC_{\text{min}} = \frac{I_{\text{max}}}{2} (H) + \frac{D}{Q_0} (S) \)

- \( H \): carrying cost
- \( D \): annual demand

Cycle Time: \( \frac{Q_0}{u} \)

Run Time: \( \frac{Q_0}{p} \)

Example 4a: A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series. The firm makes its own wheels, which it can produce at a rate of 800 per day. The toy trucks are assembled uniformly over the entire year. Carrying cost is $1 per wheel per year. Setup cost for a production run of wheels is $45. The firm operates 240 days per year. Determine the optimal run size.
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Non-instantaneous - Example

\[ p = \text{production or delivery rate} = 800 \text{ wheels per day} \]
\[ D = \text{annual demand} = 48,000 \text{ wheels} \]
\[ u = \text{usage rate} = \frac{48,000}{240} = 200 \text{ wheels per day} \]
\[ S = \text{ordering cost} = $45 \]
\[ H = \text{carrying cost} = $1 \text{ per wheel per year} \]

Economic Run Quantity

\[ Q_0 = \sqrt{\frac{2(pD)}{H}} - \frac{1}{u} \sqrt{\frac{800}{800 - 200}} \]
\[ = \frac{2(48,000)(45)}{1} \sqrt{\frac{800}{800 - 200}} \]
\[ = 2,400 \text{ wheels} \]

Example 4b: Compute the minimum total cost for carrying and setup.

\[ TC_{\text{min}} = \text{Carring Cost} + \text{Setup Cost} \]
\[ = \frac{I_{\text{max}}}{2}(H) + \frac{D}{Q_0}(S) \]
\[ = \frac{1,800}{2}(1) + \frac{48,000}{2,400}(45) \]
\[ = 900 + 900 = $1,800 \]

Example 4c: Compute the cycle time for the optimal run size.

\[ \text{Cycle Time} = \frac{2,400}{200} = 12 \text{ days} \]
thus a run of wheels will be made every 12 days

Example 4d: Compute the run time for the optimal run size.

\[ \text{Run Time} = \frac{2,400}{800} = 3 \text{ days} \]
thus each production of wheels will take 3 days
EOQ – Non-Instantaneous Replenishment

EOQ With Quantity Discount

EOQ with Quantity Discount is very important because price reductions are frequently offered to induce customers to order in larger quantities.

Why do you think this is done?

In this model, the purchasing cost will vary depending on the quantity purchased. Purchasing cost was omitted in the previous EOQ models because the price per unit was the same for all units; thus, the inclusion of the purchase cost would only increase the total cost function by the purchase cost amount. Thus, it would have had no effect on the EOQ calculation. This is illustrated in the next slide.
In this model, how much to order depends on purchase costs, setup costs and carrying costs. The Economic Order Quantity (EOQ) is the order size that minimizes the total cost of inventory. Sometimes this is referred to as the Economic Run Quantity because it is dependent on the cumulative manufacturing production quantity.

Total cost = Carrying costs + Ordering costs + Purchasing Cost

\[
TC = \frac{Q}{2} H + \frac{D}{Q} S + PD \quad \text{where } P = \text{unit price}
\]

The Method of Computing EOQ with Quantity Discount is a step wise process.

First, compute the common EOQ using the earlier formula.

Second, identify the price range where the common EOQ lies.

If the common EOQ is in the lowest quantity range then the EOQ with quantity discount is the common EOQ.

Otherwise, the EOQ with quantity discount is the quantity where the total cost is minimum when considering the cost for the common EOQ and the cost for all minimum quantities of price breaks greater than the common EOQ.
Example 5a: The maintenance department of a large hospital uses 816 cases of liquid cleanser annually. Ordering costs are $12, carrying costs are $4 per case per year, and the price schedule for ordering is listed below. Determine the optimal order quantity and the total cost. There are 240 days in a year.

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Price Per Box</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 49</td>
<td>20.00</td>
</tr>
<tr>
<td>50 to 79</td>
<td>18.00</td>
</tr>
<tr>
<td>80 to 99</td>
<td>17.00</td>
</tr>
<tr>
<td>100 or more</td>
<td>16.00</td>
</tr>
</tbody>
</table>

\[ D = \text{annual demand} = 816 \text{ cases/ year} \]
\[ S = \text{ordering cost} = \$12 \]
\[ H = \text{carrying cost} = \$4 \text{ per case per year} \]

**Common EOQ**

\[ Q = \sqrt{\frac{2DS}{H}} \]
\[ = \sqrt{\frac{2(816)(12)}{4}} = 70 \text{ cases} \]

Common EOQ = 70 cases

Is in the second price break

Therefore, the

**EOQ with quantity discount = minimum (TC(70), TC(80), TC(100))**
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EOQ With Quantity Discount - Example

\[
TC_{100} = \frac{70}{2} (4) + \frac{816}{70} \cdot 12 + 18(816) = $14,968 \\
TC_{80} = \frac{80}{2} (4) + \frac{816}{80} \cdot 12 + 17(816) = $14,154 \\
TC_{100} = \frac{100}{2} (4) + \frac{816}{100} \cdot 12 + 16(816) = $13,354
\]

Therefore; the total cost is minimum for an order quantity of 100 and the EOQ with quantity discount = 100
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The **Reorder Point (ROP)** is the quantity of inventory on hand that triggers a reorder. Four determinants for a reorder point:

- Rate of demand (based on a forecast)
- Lead time
- Extent of demand and lead time variability
- Degree of stockout risk acceptable

The ROP calculation will depend on the variability situation:

- Variability in demand
- Variability in lead time
- Variability in demand & lead time

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**When To Reorder**

If demand and lead time are both constant, the Reorder Point (ROP) can be calculated by the following formula:

\[ ROP = d \times LT \]

where

\( d \) = demand rate (units per day or week)

\( LT \) = lead time in days or weeks

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**ROP - Constant Demand & Lead Time**

Example 7: Tingly takes a “Two-A-Day” vitamins, which are delivered to his home by a route man seven days after the order is called in. At what point should Tingly reorder?

Demand rate \( d \) = 2 vitamins per day

Lead time \( LT \) = 7 days

\[ ROP = 2 \times 7 = 14 \text{ vitamins} \]
If variability in demand or lead time is present the ROP is calculated using the following general formula. 3 specific formula will follow depending upon what is variable (demand, lead time, demand & lead time):

\[ \text{ROP} = \text{Expected demand during lead time} + \text{Safety Stock} \]

Safety Stock - stock that is held in excess of expected demand due to the variability in demand rate and/or lead time.

For example: If the expected demand during lead time is 100 units and the desired amount of safety stock is 10 units then

\[ \text{ROP} = 100 + 10 \]

It is rarely the case in business where demand & lead time are constant. Variability can exist because of many reasons (customers, transportation, etc.); therefore, we must consider these impacts on inventory.

The calculation of safety stock depends on the variability of demand, lead time and the service level the organization desires.

Service Level – is the proportion of customer orders that are serviced on-time. Customers usually understand that 100% of their orders will not be serviced on-time and will establish standards for service.

By developing a probability distribution of demand during lead time, a company can use statistical calculations which determine how much safety stock is necessary to meet customer service requirements. In this case, the supply of inventory on hand a company must have to meet customer requirements is calculated by

\[ \text{supply (inventory on hand)} = \text{expected demand} + \text{safety stock} \]

This is depicted on the next slide.
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When variability in demand and/or lead time are present, and the standard deviation of the demand during lead time can be calculated, Safety Stock is calculated using the following formula:

\[ \text{Safety Stock} = z \sigma_{dLT} \]

- \( z \) = number of standard deviations necessary to achieve the desired service level
- \( \sigma_{dLT} \) = the standard deviation of lead time demand

Example 8: Suppose that a manager of a construction supply house determined from historical records that demand for sand during lead time averages 50 tons. In addition, suppose the manager determined that demand during lead time could be described by a normal distribution that has a mean of 50 tons and a standard deviation of 5 tons. Answer the following questions assuming the manager is willing to accept a stockout risk of no more than 3%.

Example 8a: What value of \( z \) is appropriate?

The risk of a stockout is .03; therefore, the service level (probability of no stockout) is .97. We can look this up in the standard normal distribution tables to calculate this number.

\[ Z(97\%) = 1.881 \]
Example 8b: How much safety stock should be held?

\[ \text{Safety Stock} = z \sigma_{LT} = 1.88(5) = 9.40 \text{ tons} \]

Example 8c: What reorder point should the manager use?

\[ \text{ROP} = \text{Expected demand during lead time} + \text{Safety Stock} \]
\[ = 50 + 9.40 = 59.40 \text{ tons} \]

Other Considerations

In the previous example, we were given the demand during lead time. When data on lead time demand are not readily available, we must determine the demand during lead time (which will depend on where variability exists). In this case there are 3 different formulas for calculating the reorder point (ROP). The formula will depend on the variability situation.

- Only variability in demand
- Only variability in lead time
- Both demand & lead time are variable

ROP – Variability in Demand

If only demand is variable then the following formula is used to calculate the ROP.

\[ \text{ROP} = \text{Expected demand during lead time} + \text{Safety Stock} \]
\[ = \bar{d}(LT) + z \sigma_{LT} \] where
\[ \bar{d} = \text{average daily or weekly demand} \]
\[ \sigma_{LT} = \text{standard deviation of the demand per day or week} \]
\[ LT = \text{lead time in days or weeks} \]
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ROP – Variability In Lead Time
If only lead time is variable then the following formula is used to calculate the ROP.

\[ \text{ROP} = \text{Expected demand during lead time} + \text{Safety Stock} \]
\[ = d \times (\text{average Lead Time}) + z \times \sigma_{LT} \]
where
\[ d = \text{daily or weekly demand} \]
\[ \sigma_{LT} = \text{standard deviation of the lead time in days or weeks} \]

ROP – Variability In Demand & Lead Time
If both lead time and demand are variable then the following formula is used to calculate the ROP.

\[ \text{ROP} = \text{Expected demand during lead time} + \text{Safety Stock} \]
\[ = \bar{d}(\text{average Lead Time}) + z \sqrt{\text{average Lead Time}} \times \sigma_{d}^2 + d \times \sigma_{LT}^2 \]
where
\[ \bar{d} = \text{average daily or weekly demand} \]
\[ \sigma_{d}^2 = \text{variance of demand in days or weeks} \]
\[ \sigma_{LT}^2 = \text{variance of lead time in days or weeks} \]

Inventory Management
As you can see, inventory management can be fairly complicated because of all of the scenarios that are possible. Using the correct quantitative tools to manage this very critical component of a business “cash flow” can reap great rewards.

The process even becomes more complicated when we realize that an end product (dependent demand) is made up of components (independent demand) and that inventory must be managed at the component level as well as the end product level.

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Homework
- Read and understand all material in the chapter.
- Discussion and Review Questions
- Recreate and understand all classroom examples
- Exercises on chapter web page