Lesson 14 – Statistical Process Control

**Purpose**

The purpose is to assure that processes are performing in an acceptable manner.

![Chart showing control limits and sample numbers](chart.png)

**Quality Assurance – An Evolutionary Process**

Developing an effective quality assurance program is an evolutionary process beginning with the quality assurance of incoming materials ... evolving to total quality management.

![Diagram showing least progressive to most progressive process control](diagram.png)

**Quality Control Concepts**

The manufacturing manager of the Hiney Winery is responsible for filling the Tiny Hiney wine bottles with 16 oz of wine. If the wine bottles are too full the Hiney Winery loses money and if they are do not contain 16 ounces their customers get upset.

The steps involved in filling the Heiney Wine bottles is called a process. The process involves people, machinery, bottles, corks, etc.

Do you think all Tiny Hiney’s will contain exactly 16 ounces of wine? Why or Why Not?

![Image of Tiny Hiney bottles](image.png)
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Which sample has less variability?

16 oz.  
16 oz.

We measure process variability based on a sample statistic (mean, range, or proportion). The goal of statistical process control is to monitor and reduce process variability.

All processes will vary over time!

What causes a process to vary?

People  
Machines  
Materials  
Methods  
Measurement  
Environment

Random or Natural Variation (In Control)

Special or Assignable Variation (Out of Control)

Can you think of an example of random and assignable variation for each of the causes?

Can you think of a way to reduce it?
Random or Assignable Variation? Why?

- Assignable Variation
  - Correctable problems
  - Not part of process design
  - Due to machine wear, unskilled workers, poor material, etc.
  - Can only be detected in a process which has stable or constant variation. The goal of process quality control is to identify assignable causes of variation and eliminate them. Random variation is much harder to identify; however, the goal remains to improve variability.

Monitoring Production (Inspection)

- It is often impossible, impractical and cost prohibitive to inspect each and every element of production therefore statistical methods based on probability of failure are necessary to achieve a cost effective method of assuring quality.
  - Inspection of inputs/outputs: Acceptance Sampling
  - Inspection during the transformation process: Process Control
  - Acceptance Sampling
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Issues Involved In Inspection

How much? How Often?

. Where (at what points) should inspection occur?
. Centralized or On-site inspection?

The traditional view of the amount of inspection was that there was an optimum (cost) amount of inspection necessary to minimize risk of passing a defective. This was predicated on the facts:

. As the amount of inspection increases the defectives that “get through” will decrease (i.e., the “cost of passing defectives goes down”)
. As the amount of inspection increases the cost of inspection increases

The current view is that every effort involved in reducing the number of defectives will reduce costs.

Traditional View – How Much To Inspect

Cost of inspection (goes up as amount increases)

Cost of passing defectives (goes down as amount increases)

Total Cost

Optimal Amount of Inspection

Issues Involved In Inspection

How much to inspect and how often the inspection is done depends

. On the item
  .. Low cost, high volume items - little inspection
     (e.g. paper clips, pencils, glassware, etc)
  .. High cost, low volume items with a high cost of a passing defective - more inspection (e.g. space shuttle)

. On the processes that produce the item
  .. Reliability of the equipment
  .. Reliability of the human element
  .. The stability of the process ... stable processes (those that infrequently go “out of control”) require less inspection than unstable processes

. On the lot (batch) size ... large volumes of small lots will require more inspection than small volumes of large lots
Where to inspect depends on:
- Raw materials and purchased parts
- Finished products
- Before a costly operation (don’t add costs to defective items)
- Before a covering process such as painting or plating
- At points where there is a high variability in the output either as a result of mechanical or human variability
- Before an irreversible process

Centralized or On-site Inspection decisions are important
- Centralized makes sense in retail environments, medical environments, when specialized testing equipment is required
- On-site makes sense when quicker decisions are necessary to ensure processes (mechanical/human) remain in control

<table>
<thead>
<tr>
<th>Type of business</th>
<th>Inspection points</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast Food</td>
<td>Cashier, Counter area, Eating area, Building, Kitchen</td>
<td>Accuracy, Appearance, productivity, Cleanliness, Appearance, Health regulations</td>
</tr>
<tr>
<td>Hotel/motel</td>
<td>Parking lot, Accounting, Building, Main desk</td>
<td>Safe, well lighted, Accuracy, timeliness, Appearance, safety, Waiting times</td>
</tr>
<tr>
<td>Supermarket</td>
<td>Cashiers, Deliveries</td>
<td>Accuracy, courtesy, Quality, quantity</td>
</tr>
</tbody>
</table>
Statistical Process Control

Statistical Process Control (SPC) is a methodology using statistical calculations to ensure a process is producing products which conform to quality standards (design based, manufacturing based, customer based).

Major activities:
- Monitor process variation
- Diagnose causes of variation
- Eliminate or reduce causes of variation

Designed to keep or bring a process into statistical control.

Statistical Process Control involves statistical evaluation of a process throughout the whole production cycle. The control process requires the following steps:
- Define in sufficient detail the characteristics to be controlled, what “in” and “out” of control means (i.e., What is the quality conformance standard?)
- Measure the characteristic (based on sample size data)
- Compare it to the standard
- Evaluate the results
- Take corrective action if necessary
- Evaluate the corrective action

Statistical Process Control Steps

Start → Produce Good or Service → Take Sample → Inspect Sample → Create/Update Control Chart → Special Causes
- Yes → Stop Process
- No → Find Out Why
All processes possess exhibit a natural (random) variability which are produced by a number of minor factors. If any one of these factors could be identified and eliminated, the change to the natural variability would be negligible.

An assignable variability is where a factor which contributes to the variability of a process can be identified and quantified. Usually this type of variation can be eliminated thus reducing the variability of the process.

When the measurements exhibit random patterns the process is considered to be in control. Non-random variation typically means that a process is out of control.

Process Variability can be measured and a probability distribution can be calculated … it might look something like this

Which process exhibits more stability (less variation)?

When the factors influencing process variability can be identified and eliminated the probability distribution will exhibit less variability.

The Central Limit Theorem states that (for large samples) the sampling distribution is approximately Normal regardless of the process distribution; therefore, the Normal Distribution can be used to evaluate measurement results and to determine whether or not a process is “in control”.

As the sample size increases, the variation in the sampling distribution decreases.
The standard deviation (a measure of variability) of the sampling distribution is key to the statistical process control methods. 3 standard deviations (99.7% confidence) are typically used to establish Control Limits (LCL, UCL) used to monitor process control.

In statistical process control, we are concerned with the sampling distribution. Control Limits (LCL, UCL) are set using the standard deviation of the sampling distribution.

Control Limits

In Control or Out of Control?

Sample measurements between the LCL and UCL usually indicate that a process is in control. The probability of concluding a process is out of control when it is actually in control is called a Type I error. The amount of probability (α) left in the tail is referred to as the probability of making a Type I error.
A Process Control Chart is a time series showing the sample statistics and their relationship to the Lower Control Limit (LCL) and Upper Control Limit (UCL). Once a control chart is established it can be used to monitor a process for future samples. The Process Control Chart may need to be re-evaluated if improvements are made.

We expect to see normal or random variation due to chance for measurements within the control limits. Measurements outside the control limits may indicate abnormal variation usually due to assignable sources.

We expect to see normal or random variation patterns within the control limits; however, this does not guarantee a process is in control. Consider the following example. What should a quality control manager do in this situation?
In Control or Out of Control

A process is deemed to be in control if the following two conditions are met.

1. Sample statistics are between the control limits
   Control Charts are used to test this condition
2. There are no non-random patterns present
   Pattern analysis rules are used to test this condition

Otherwise, the process is deemed to be out of control.

Control Charts

Four commonly used control charts

- Control charts for variables (measured or continuous)
  - Range chart - monitor variation (dispersion)
  - Mean charts - monitor central tendency
- Control charts for attributes (counted or discrete)
  - p-chart - used to monitor the proportion of defectives generated by a process
  - c-chart - used to monitor the number of defects per unit (e.g. automobiles, hotel rooms, typed pages, rolls of carpet)

Pattern Analysis

Run Charts – Check For Randomness

Used in conjunction with control charts to test for randomness of observational data. The presence of patterns in the data or trends indicates that non-random (assignable) variation is present.

A run is a sequence of observations with a certain characteristic.

Two useful run tests:
- Runs above and below the centerline
  data translated into A (above) and B (below)
- Runs up and down
  data translated into U (up) and D (down)
Counting A/B Runs

Counting Above/Below Runs

(7 runs)

If a value is equal to the centerline, the A/B rating is different from the last A/B rating.

Counting U/D Runs

Counting Up/Down Runs

(8 runs)

Note: the first value does not receive a notation.

If two values are equal, the U/D rating is different from the last U/D rating.

Expected Number of Runs

Once the runs are counted, they must be compared to the expected number of runs in a completely random series. The expected number of runs and the standard deviation of the expected number of runs are computed by the following formula:

\[ E(r)_{a/b} = \frac{N}{2} + 1 \]

\[ \sigma_{a/b} = \sqrt{\frac{N - 1}{4}} \]

\[ E(r)_{u/d} = \frac{2N - 1}{3} \]

\[ \sigma_{u/d} = \sqrt{\frac{16N - 29}{90}} \]

N = number of observations
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**Compare Observed Runs to Expected Runs**

Next, we compare the observed number of runs to the expected number of runs by calculating the following **Standard Normal Z-statistic**:

\[
Z_{a/b} = \frac{r - E(r)_{a/b}}{\sigma_{a/b}}
\]

\[
Z_{u/d} = \frac{r - E(r)_{u/d}}{\sigma_{u/d}}
\]

\[r = \text{observed number of runs}\]

**Run Test - In or Out of Control**

For a degree of confidence of 99.7% (3 standard deviations), we compare the comparison to -3 standard deviations and +3 standard deviations.

If either the \(Z_{a/b}\) or \(Z_{u/d}\) is

- **< lower limit** then we have too few runs (Out of Control)
- **between the lower limit and upper limit** then we have an acceptable number of runs (In Control)
- **> upper limit** then we have too many runs (Out of Control)

**Statistical Process Control**

- **Control Charts**
  - **Quantitative Variable Charts**
    - Range Chart
    - Mean Chart
  - **Qualitative Attribute Charts**
    - P Charts
    - C Charts
The Goodman Tire & Rubber Company periodically tests its tires for tread wear under simulated road conditions. To monitor the manufacturing process 3 radial tires are chosen from each shift of production operation. The operations manager recently attended a statistical process control (SPC) seminar and decided to adopt SPC to control and study tread wear.

The manager believes the production process is under control and to establish control limits and monitor the process he took 3 randomly selected radial tires from 20 consecutive shifts. They were tested for tread wear and recorded in the table on the next slide.

Tread wear is recorded in millimeters which is a measured variable; therefore, a range and mean chart should be developed.

Let’s take a look at the steps the manager must use to establish and analyze the control chart.

First of all, he notes:
- sample size = 3
- process variability is not known
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### 3-Sigma Control Chart Table

Next we must determine the center line, upper and lower control limits for the range and mean chart which are calculated by very simple formula. There is only one formula for the range chart. However, there are two formulae for the mean chart. The one chosen is dependent on whether the variability (standard deviation) of the process is known or unknown.

The 3-sigma control chart table is used to calculate these formulae.

### Range Chart (R Chart)

The Range Chart measures the variability of a sample measurements in a process. The range chart must be interpreted first because the process variability must be in control before we can monitor whether a process is in or out of control. The Range Chart Centerline, UCL, and LCL are calculated by the following formula.

- **Centerline** = grand range = $\overline{R}$
- **LCL** = $\overline{R} \times D_1$
- **UCL** = $\overline{R} \times D_3$

The Goodman Tire & Rubber Company range chart calculations are:

- **Centerline** = 1.14
- **LCL** = 1.14 * 0
- **UCL** = 1.14 * 2.574
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**Control Chart**

Centerline, 1.1400
LCL, 0.0000
UCL, 2.9344

**Mean Chart (X̄ Chart) - Known Variation**

The Mean Chart measures the average of sample measurements in a process. When the variation of the process is known, the 3-sigma table is not used.

The Mean Chart Centerline, UCL and LCL (3-sigma limits) are calculated by the following when the variation of the process is known.

Centerline = grand average = \( \bar{X} \)

\[
LCL = \bar{X} - 3 \frac{\sigma}{\sqrt{n}}
\]

\[
UCL = \bar{X} + 3 \frac{\sigma}{\sqrt{n}}
\]

where : \( n \) = sample size

\( \sigma \) = process standard deviation

**Mean Chart (X̄ Chart) - Unknown Variation**

The Mean Chart measures the average of sample measurements in a process. When the variation of the process is unknown, the 3-sigma table is used.

The Mean Chart Centerline, UCL and LCL are calculated by the following formula when the variation of the process is unknown.

Centerline = grand average = \( \bar{X} \)

\[
LCL = \bar{X} - A_2 \bar{R}
\]

\[
UCL = \bar{X} + A_2 \bar{R}
\]

The Goodman Tire & Rubber Company mean chart calculations are:

Centerline = 2.9167

\[
LCL = 2.9167 - 1.14 * 1.023
\]

\[
UCL = 2.9167 + 1.14 * 1.023
\]
The sample mean values are now plotted on the mean control chart.

1. Choose the RM (Range & Mean) menu option.
2. Enter Data.
3. Select Known or Unknown.
4. Return.
5. Enter problem name then click OK.

6. Choose the CC (Control Chart) menu item.

7. Choose the Range control chart.

Notice the range control chart shows the sample ranges.

Pattern Analysis for AB and UD indicate the patterns are random (YES).

Therefore, the range chart indicates the variation of the process is in control.
8. Choose the Mean Unknown Variation control chart

Notice the mean control chart shows the sample averages.
Pattern Analysis for Ad and Ud indicate the patterns are random (YES)
Therefore, the mean chart indicates the average of the process is in control.

Both charts are necessary to effectively monitor a process.
The mean charts detect when a process mean is shifting.
The range charts detect when the process variability is changing.
The range chart must be interpreted first. If the variability is not constant, the process is not in control.

Summary - Range & Mean Control Charts

Sampling Distribution
R-Chart
Detects shift in process mean

(process mean is shifting upward)

R-chart
Does not detect shift
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### Mean & Range Control Charts

- **Sampling Distribution**
  - (process variability is increasing)

- **\( \bar{x} \)-Chart**
  - Does not reveal increase

- **R-chart**
  - Reveals increase

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### Statistical Process Control

#### Control Charts

- **Quantitative Variable Charts**
  - Range Chart
  - Mean Chart

- **Qualitative Attribute Charts**
  - P-Chart
  - C Chart

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### P-Chart (Proportion Chart)

The P-Chart measures the proportions in a process.

The Proportion Chart Centerline, UCL and LCL are calculated by the following formula.

- **Centerline** = \( \hat{p} \)
- **LCL** = \( \hat{p} - 3 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \)
- **UCL** = \( \hat{p} + 3 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \)

- \( \hat{p} \) = total number defective
- \( n \) = number of samples
- \( n \) = sample size

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14-19
Automated mail sorting machines are used to sort mail by zip codes. These machines scan the zip codes and divert each letter to its proper carrier zone.

Even when a machine is properly operating some mail is improperly diverted (a defective).

Check the sorting machine here to see if it is in control.

### Mail Sorting

<table>
<thead>
<tr>
<th>Sample</th>
<th>Sample Size</th>
<th>Defective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>12</td>
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<tr>
<td>11</td>
<td>100</td>
<td>11</td>
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<td>10</td>
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<td>10</td>
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<tr>
<td>15</td>
<td>100</td>
<td>11</td>
</tr>
<tr>
<td>16</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>17</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>18</td>
<td>100</td>
<td>9</td>
</tr>
<tr>
<td>19</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
<td>16</td>
</tr>
</tbody>
</table>

Calculate the actual proportion defective per sample by dividing the observed defective by the sample size.

The average proportion is

\[ \hat{p} = \frac{14 + 10 + 12 + 13 + 12 + 11 + 9 + 13 + 10 + 12 + 11 + 9 + 10 + 10 + 11 + 10 + 12 + 9 + 10 + 12 + 16}{20 	imes 100} = 0.0161 \]

LCL = 0.1100

Centerline = 0.2039

UCL = 0.3076

The sample proportion values are now plotted on the proportion control chart.
Notice the proportion control chart shows the sample proportions. Pattern Analysis for AB and UD indicate the patterns are random (YES).
Therefore, the proportion chart indicates the process is in control.

Types of Control Charts

- Control Charts
  - Quantitative Variable Charts
  - Qualitative Attribute Charts
    - Range Charts
    - Mean Charts
    - P Charts
    - C Charts

C-Chart (Defects Per Unit)
The C-Chart is used to monitor the number of defects per unit (e.g., automobiles, hotel rooms, typed pages, rolls of carpet).
The C Chart Centerline, UCL and LCL are calculated by the following formulas:

- UCL = \( \bar{c} + 3\sqrt{\bar{c}} \)
- LCL = \( \bar{c} - 3\sqrt{\bar{c}} \) where
- \( \bar{c} \) = process average number of defects per unit

In the event \( \bar{c} \) is unknown, it can be estimated by sampling and computing the average defects observed. In this case the average can be substituted in the formula for \( \bar{c} \). Also, note that since the formula is an approximation the LCL can be negative. In these cases the LCL is set to 0.
Example 5: Rolls of coiled wire are monitored using a c-chart. Eighteen rolls of wire have been examined and the number of defects per roll has been recorded in the data provided here. Is the process in control? Use 3 standard deviations for the control limits.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Defects</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
</tr>
</tbody>
</table>

The average number of defects per roll of coiled wire is

\[ \bar{c} = 2.5 \]

The sample number defects values are now plotted on the defects per unit control chart.

3σ Control Chart Settings

- UCL: 7.2564
- Centerline: 2.5000
- LCL: 0.0000

The sample number defects values are now plotted on the defects per unit control chart.
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Choose the Defects per unit (C) control chart

Notice the unit defective control chart shows the sample proportions.

Pattern Analysis for AB and UD indicate the patterns are random (YES)

Therefore, the C chart indicates the process is in control

Process variability can significantly impact quality. Some commonly used terms refer to the variability of a process output.

- **Tolerances** - specifications for the range of acceptable values established by engineering or customer specifications.
- **Control limits** - statistical limits that reflect the extent to which sample statistics such as means and ranges can vary due to randomness.
- **Process variability** - reflects the natural or inherent (e.g., random variability in a process. It is measured in terms of the process standard deviation.
- **Process capability** - the inherent variability of process output relative to the variation allowed by the design specification.

3-sigma control is when the process performs within 3 standard deviations of the mean.

6-sigma control is when the process performs within 6 standard deviations of the mean.

Three Sigma and Six Sigma Quality

<table>
<thead>
<tr>
<th>Product Lower specification</th>
<th>Product Upper specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7 ppm</td>
<td>1.7 ppm</td>
</tr>
<tr>
<td>1.35 ppt</td>
<td>1.35 ppt</td>
</tr>
</tbody>
</table>

ppt – parts per thousand
ppm – parts per million

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Example 7: A manager has the option of using any one of the following three machines for a job. The machines and their standard deviations are listed below. Determine which machines are capable of doing the job if the specifications are 1.00mm to 1.60mm.

<table>
<thead>
<tr>
<th>Machine</th>
<th>Std Dev (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.10</td>
</tr>
<tr>
<td>B</td>
<td>0.08</td>
</tr>
<tr>
<td>C</td>
<td>0.13</td>
</tr>
</tbody>
</table>

The specifications indicate that the tolerance width is 0.6mm. (e.g. 1.60 – 1.00)

Process capability for 3-sigma control is deemed to be within 3 standard deviations of the mean. Therefore for this example the process capability is 6 * (standard deviation) shown in the table below.

<table>
<thead>
<tr>
<th>Machine</th>
<th>Std Dev (mm)</th>
<th>Machine Capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.10</td>
<td>0.60</td>
</tr>
<tr>
<td>B</td>
<td>0.08</td>
<td>0.48</td>
</tr>
<tr>
<td>C</td>
<td>0.13</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Therefore, Machine A & B are capable of meeting the specifications, Machine C is not.
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Homework

Read and understand all material in the chapter.
Discussion and Review Questions
Recreate and understand all classroom examples
Exercises on chapter web page