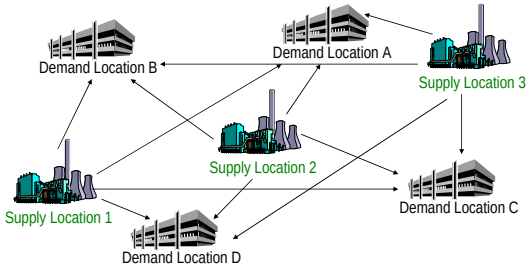
Lesson 12
The Transportation Model

An Application of Linear Programming

12 - 1

The Problem

Minimize transportation costs between *Supplying Locations (Factories)* and *Demand Locations (Warehouses).*



12 - 2

The Cost Matrix

The transportation *cost per unit* between *Factory* and *Warehouse* are shown in the cost matrix table below:

	Warehouse			
Factory	A	B	C	D
1	4	7	7	1
2	12	3	8	8
3	8	10	16	5

The transportation cost to ship 1 unit of product between Factory 1 and Warehouse A is 4.

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Factory Capacity & Warehouse Demand

The *factory capacity (supply units per period)* and the *warehouse demand (units per period that can be handled)* are shown in the following tables:

Factory	Supply
1	100
2	200
3	150
Total	450

	Warehouse				
	A	B	C	D	Total
Demand	80	90	120	160	450

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Summary Matrix

The three previous tables can be summarized in one matrix as follows:

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	1	100	
2	12	3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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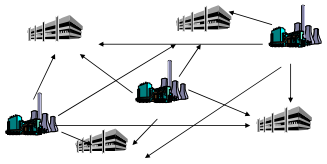
An Optimal Solution

Once the initial allocation is made, there are methods for obtaining an optimal solution which involve still further steps - these are discussed in the section "*Testing for Optimality*"*pages 393 though 405.*

Although, the manual solution to the transportation model is relatively straightforward, it is time consuming. *The transportation model can also be optimally solved by Linear Programming.*



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The LP Formulation

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	1	100	
2	12	3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

let $x_{i,j}$ be the quantity shipped from factory i to warehouse j

minimize $4x_{1,A} + 7x_{1,B} + 7x_{1,C} + 1x_{1,D} +$
$12x_{2,A} + 3x_{2,B} + 8x_{2,C} + 8x_{2,D} +$
$8x_{3,A} + 10x_{3,B} + 16x_{3,C} + 5x_{3,D}$

12 - 7

The LP Formulation

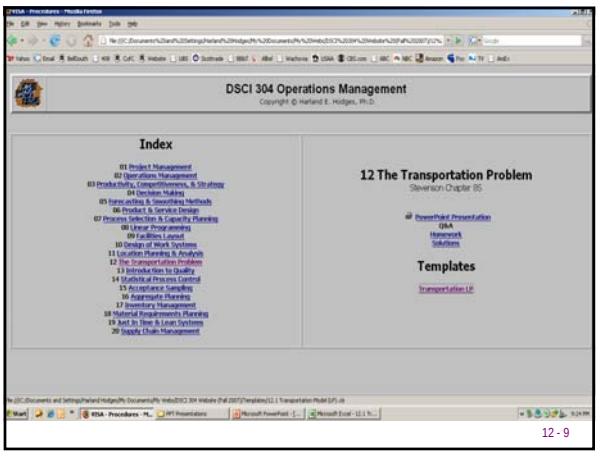
. *Supply Constraints (rows)*

subject to $x_{1,A} + x_{1,B} + x_{1,C} + x_{1,D} = 100$
$x_{2,A} + x_{2,B} + x_{2,C} + x_{2,D} = 200$
$x_{3,A} + x_{3,B} + x_{3,C} + x_{3,D} = 150$

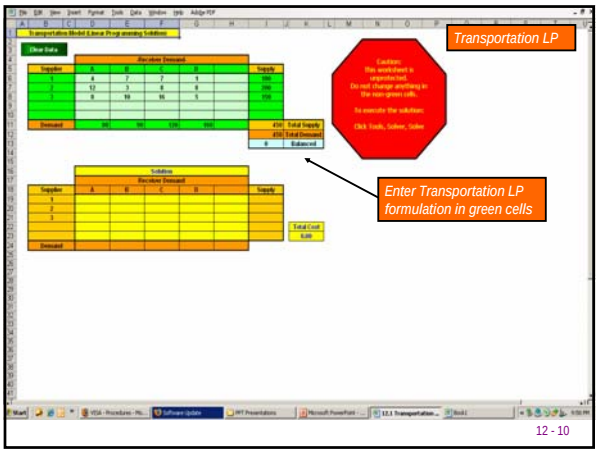
. *Demand Constraints (columns)*

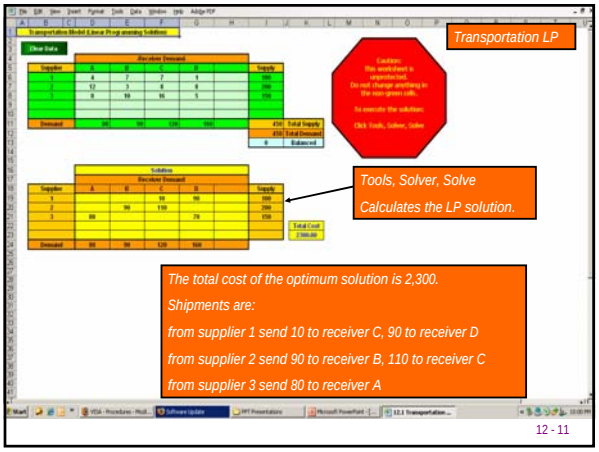
subject to $x_{1,A} + x_{2,A} + x_{3,A} = 80$
$x_{1,B} + x_{2,B} + x_{3,B} = 90$
$x_{1,C} + x_{2,C} + x_{3,C} = 120$
$x_{1,D} + x_{2,D} + x_{3,D} = 160$

12 - 8



12 - 9





Unequal Supply & Demand

Consider the following situation showing cost per unit between supply and demand (receiving) location where the *supply* and the *demand* are unequal.

	Demand		
Supply	A	B	Supply
1	9	6	75
2	5	3	75
	80	90	Total Supply
	Total Demand	170	150

In this case the ability of the *demand* (receiving) locations is *20* more than the *supply* locations.

12 - 12

Unequal Supply & Demand

In this case we must balance the supply and demand by introducing a *Dummy Supply Location for 20 units*.

	Demand			
Supply	A	B		Supply
1	9	6		75
2	5	3		75
Dummy				20
	80	90		Total
	Total Demand		170	Supply
				170

Note: No cost is entered for the *Dummy* location. Now, the problem can be solved using the Linear Programming solution for the Transportation Problem as shown on the next slide.

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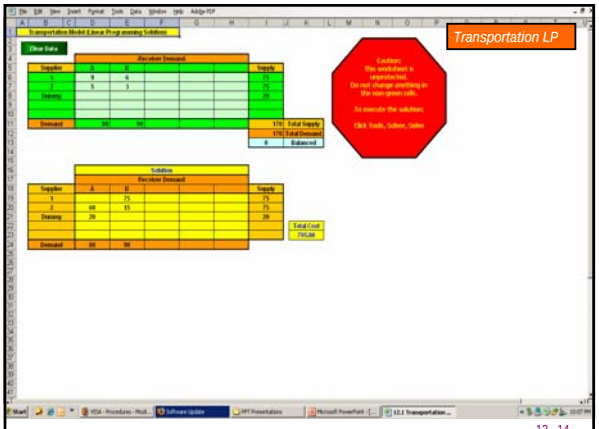

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Evaluating Alternatives

Example: A company that specializes in nonferrous casting currently has 3 Warehouses (receiving locations) and two casting foundry factories (supply locations). The shipping cost, factory capacity and warehouse capacity are summarized in the following table.

	Warehouse			
Foundry	A	B	C	Supply
1	17	10	6	30
2	7	12	14	20
Demand	25	10	40	Total
	Total Demand		75	Supply
				50

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Evaluating Alternatives

Business conditions have been good and the company will build a new foundry to meet future business requirements. They are considering 2 locations: Chicago and Detroit. The new foundry will be designed to produce 2,500 nonferrous casts per month. The new foundry will be shipping their product to the current warehouses. The accounting department has determined the shipping costs from Detroit and Chicago to the existing warehouses and summarized them in the table below:

	Warehouse		
New Foundry	A	B	C
Detroit	10	8	15
Chicago	12	13	5

Which location should they choose? Detroit or Chicago?

12 - 16

Evaluating Alternatives

To answer this question we must consider the Transportation Problem Solution for both Detroit and Chicago then evaluate the results:

Detroit – Transportation Problem

	Warehouse Supply			
Factory Demand	A	B	C	Supply
1	17	10	6	30
2	7	12	14	20
Detroit	10	8	15	25
Demand	25	10	40	Total Supply
	Total Demand		75	75

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Evaluating Alternatives

Detroit – Solution

	Warehouse Supply			
Factory Demand	A	B	C	Supply
1	0	0	30	30
2	20	0	0	20
Detroit	5	10	10	25
Demand	25	10	40	Total Supply
	Total Demand		75	75

Total Cost	600

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Evaluating Alternatives

Chicago – Transportation Problem

	Warehouse Supply			
Factory Demand	A	B	C	Supply
1	17	10	6	30
2	7	12	14	20
Chicago	12	13	5	25
Demand	25	10	40	Total Supply
	Total Demand		75	75

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Evaluating Alternatives

Best

Chicago – Solution

	Warehouse Supply			
Factory Demand	A	B	C	Supply
1	17	10	6	30
2	7	12	14	20
Chicago	12	13	5	25
Demand	25	10	40	Total Supply
	Total Demand		75	75

Total Cost	520

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Homework

Read and understand all material in the chapter.

Discussion and Review Questions

Recreate and understand all classroom examples

Exercises on chapter web page

12 - 21

Appendix: A Heuristic Solution

A *heuristic* (intuitive) argument can be made for the solution to this problem. It follows the steps below:

. Identify the cell with the lowest cost
. Allocate as many units as possible to that cell and cross out the row or column (or both) that is exhausted by this assignment
. Find the cell with the next lowest cost from among the feasible cells
. Repeat the second and third steps until all units have been allocated

12 - 22

A Heuristic Solution

Lets take a look at how the heuristic is applied to this problem

. Identify the cell with the lowest cost

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	1	100	
2	12	3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

12 - 23

A Heuristic Solution

. Allocate as many units as possible to that cell

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. And cross out the row or column (or both) that is exhausted by this assignment

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

12 - 25

A Heuristic Solution

. Find the cell with the next lowest cost from among the feasible cells

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. Allocate as many units as possible to that cell

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. And cross out the row or column (or both) that is exhausted by this assignment

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. Find the cell with the next lowest cost from among the feasible cells

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	8	10	16	5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. Allocate as many units as possible to that cell

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

12 - 30

A Heuristic Solution

. And cross out the row or column (or both) that is exhausted by this assignment

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

12 - 31

A Heuristic Solution

. Find the cell with the next lowest cost from among the feasible cells … In this case there is a tie … choose one arbitrarily.

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. Allocate as many units as possible to that cell

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	80/8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

12 - 33

A Heuristic Solution

. And cross out the row or column (or both) that is exhausted by this assignment

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	80/8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

12 - 34

A Heuristic Solution

. Find the cell with the next lowest cost from among the feasible cells

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	8	8	200	Total
3	80/8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. Allocate as many units as possible to that cell

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	110/8	8	200	Total
3	80/8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

12 - 36

A Heuristic Solution

. And cross out the row or column (or both) that is exhausted by this assignment

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	110/8	8	200	Total
3	80/8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. Find the cell with the next lowest cost from among the feasible cells

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	110/8	8	200	Total
3	80/8	10	16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. Allocate as many units as possible to that cell

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	110/8	8	200	Total
3	80/8	10	10/16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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A Heuristic Solution

. And cross out the row or column (or both) that is exhausted by this assignment … The distribution cost is 80*8 + 90*3 + 110*8 + 10*16 +100*1 + 60*5 = 2,350 per unit

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	110/8	8	200	Total
3	80/8	10	10/16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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Heuristic vs LP Solution

The Heuristic Solution … Cost = 2,350

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	7	100/1	100	
2	12	90/3	110/8	8	200	Total
3	80/8	10	10/16	60/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

The Linear Programming Optimum Solution … Cost = 2,300

	Warehouse					
Factory	A	B	C	D	Supply	
1	4	7	10/7	90/1	100	
2	12	90/3	110/8	8	200	Total
3	80/8	10	16	70/5	150	Supply
Demand	80	90	120	160		450
			Total Demand		450	

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